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Corry, Leo (IL-TLAV-HQS)

★**Distributivity-like results in the medieval traditions of Euclid's *Elements*—between geometry and arithmetic.**

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Leo Corry is known in particular for his earlier work on the history of modern (i.e., post-Dedekind) algebra, not least the well-received *Modern algebra and the rise of mathematical structures* [Sci. Networks Hist. Stud., 17, Birkhäuser, Basel, 1996; MR1391720; second edition, Birkhäuser, Basel, 2004; MR2033171].

The present, much shorter book starts from there, more precisely from the observation that under "the perspective of modern algebraic theories, in any system comprising two operations, the validity of a well-defined distributivity law typically embodies a kind of minimal requirement for the system to be mathematically interesting" (p. 1). Its theme is thus the question how similar structures are dealt with in a number of historical sources based on and often recasting the Euclidean framework. We may consider the inquiry an element of soft category theory, if that concept be permitted.

Pre-modern mathematics did not think in terms of abstract systems and operations. Instead of abstract systems it dealt with numbers and with continuous quantities (mostly geometric quantities, first of all line segments, areas and volumes). Its numbers were, basically, collections of units (for which reason the underlying implicit axiom is that *counting is meaningful*, from which follow directly the commutative as well as the associative law). Admittedly, the Middle Ages often widened the number domain so as to comprise also fractions and even roots, but under the supposition that this did not change matters. The operations considered were additions (understood as *heapings* or *mergers*), multiplications (understood in agreement with the understanding of numbers, that is, *repeated additions*), but also ratio taking and rectangle formation.

Since transformations of the approach of the *Elements* is the topic, Corry evidently starts by analyzing how distributivity-like identities are dealt with in Books II, V and VII of that work.

Heron's commentary to the *Elements* as reported by al-Nayrīzī is dealt with together with al-Nayrīzī's additions, where numerical illustrations to geometric theorems turn up together with the incorporation of certain theorems from *Elements* II in *Elements* VII.

Such themes are then analyzed in Abū Kāmil's and al-Khayyāmī's algebras; in the *Liber mahameleth*, Fibonacci's *Pratica geometrie* and his *Liber quadratorum*; in Sacrobosco's *Algorismus vulgaris*, Jordanus de Nemore's *De elementis arithmetice artis*, and Campanus's redaction of the *Elements*; and finally in Gersonides's *Sefer maaseh hoshev*, Alfonso de Valladolid's *Sefer meyasher ʿaqov*, in Elijah Mizrahi's *Sefer ha-mispar* and Komtino's *Sefer ha-ḥeshbon we ha-middot*; in the latter two, shortcuts for the multiplication of numbers are the main representatives of the distributivity-like principles.

As it was said about Corry's *Modern algebra* in a review [J. Ferreirós, Isis **100** (2009), no. 2, 412–413, doi:10.1086/605247], even this book "has a marked conceptual and epistemological bent", which is its force (however much a reader may sometimes disagree with the interpretations offered); when it comes to its mostly unsubstantiated statements about influences *of* and *on* the treatises under discussion, these are peripheral and perhaps at best categorized as opinions. *Jens Høyrup*